ANSDM converters dynamic range design process

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Abstract

The results of analytic and simulating works proved that for nonstationary sources, the delta converters with adaptive sampling expose higher coding efficiency than the former proposals, based on uniform sampling methods. The knowledge of the sampling interval range and the algorithm of the Nonuniform Sampling Delta Modulation and Adaptive Nonuniform Sampling Delta Modulation allows finding the necessary number of the sampling intervals and their values that maximizes SNR. The total dynamic range of the ANSDM modulator is the product of the dynamic range both from sampling interval and step size adaptation. Due to the high complexity of the calculations, the ANSDMsoft program was developed to support computing. All computational works were carried out using the Maple environment. Maple allows to solve complex mathematical functions and display their results in a simple way. Most importantly, it supports the LambertW function, used in the computing of NSDM or ANSDM modulators parameters. Graphic illustrations of the NSDM and ANSDM modulator dynamic range as a function of the minimum and maximum sampling frequency are presented.

Keywords: 1-bit Delta Modulation (DM), Adaptive Delta Modulation (ADM), Non-Uniform Sampling Delta Modulation (NSDM), SNR, Adaptive Non-Uniform Sampling Delta Modulation (ANSDM), Dynamic Range (DR), Lambert W function, sampling interval, oversampling ratio, Maple by Maplesoft

1. Introduction

DM codecs were chosen, by NASA for source encoding of voice in the space shuttles, because of its tolerance to channel errors. Compression properties of ADM systems are based on a rule that the A/D conversion is accomplished on the input process with removed redundancy. The investigation aimed at the improvement of analog to digital conversion efficiency proves that not all potential possibilities of differential processing have been fully utilized, so far. The concept of using adaptive time sampling to reduce the data rate of source coding is very promising, however, studied only fragmentarily. The author’s achievements, compared with the actual knowledge of the delta conversion systems, have been presented. The long-term research works [1, 2, 3] allow the author to affirm that reaching large noise immunity and high accuracy in a wide dynamic range, with the use of the uniform sampling ADPCM or 1-bit delta converters, is not possible.

Therefore it has been proposed to use the 2-parameters, 1-bit ADM modulation, in which both the step size and the sampling interval are changed. The research was aimed at elaboration of analysis methods and performance evaluation of the delta systems with step size and sampling rate adaptation (ANSDM) [4, 5]. It comprised the studies of mathematical description, computer simulations, and design methodology, as well as the proposal of electronic circuits implementation. The results of analytic and simulating works proved that for nonstationary sources, the delta converters with adaptive sampling expose higher coding efficiency than the former proposals, based on uniform sampling methods [2].

Paper organization

Tools for simulation research are presented in Section 2, design process of the ANSDM converter parameters calculating are presented in Section 3, whereas design values of the sampling interval range $K_f$ and quantization step size range $K_k$ for ANSDM modulators are presented in Section 4. Examples of determining
the internal parameters of NSDM and ANSDM modulators based on the Maple platform are shown in Section 5. Examples of the \( DR \) determination using ANSDMSoft are presented in Section 6. The overall conclusions are given in Section 7.

2. Tools for simulation research

Four basic output parameters \( SNR \), \( SNR_{\text{max}} \), \( BR_{\text{avg}} \) (Bit Rate average), \( f_{\text{avg}} \) (sampling frequency average), and \( DR \) (Dynamic Range) were analysed. In Figs. 1, 2, and 3, research possibilities of the 1-bit delta codec SYMMOD simulator are presented. The emulator (Fig. 1) allows the monitoring and sound reproduction of the input source (sine, triangle, rectangle, and speech source approximated by a white noise signal having the integrated power spectrum and Gaussian amplitude distribution), predictor waveform, and reconverted signal.

Simulation investigations, mathematical analyses [3] confirmed that in the case of the delta converters with adaptive sampling (NSDM, ANSDM), the average bit rate \( BR_{\text{avg}} \) decides about the maximum \( SNR \) value of the modulations (constant in all range of \( DR \)). In the Fig. 3, the user interface of the control program named SYMMOD which computes e.g. necessary values of the step sizes and sampling intervals is shown [4, 5, 6].

![Figure 1. Block diagram of the 1-bit delta converter emulation system „SYMMOD”](image)

![Figure 2. Input, predictor, output and data output signal of the ANSDM modulation system](image)
Program SYMMOD carries out an algorithmic simulation of delta modulations. It has an advanced module for analysing the generated signals (FFT analysis, SNR ratio), an error generation module, and a module of digital filters. The system enables bi-directional transmission of modulated digital signals via Ethernet. In the case of a PC with a sound card, it is possible to configure the stand so as to observe two-way sound transmission in real-time.

3. Design process of the ANSDM converter

For ANSDM modulation \( k = \text{var}, f_s = \text{var} \), in order to maximize SNR in some range of the input signal level \( DR \) the Abate’s conditions [1] have to be obeyed:

\[
\frac{k_{\text{max}} B_{\text{max}}}{\chi \sqrt{S^2}} = \ln(2 B_{\text{max}}) \tag{1}
\]

\[
\frac{k_{\text{min}} B_{\text{min}}}{\chi \sqrt{S^2}} = \ln(2 B_{\text{min}}) \tag{1'}
\]

where:

\( B = \frac{f_s}{2fc} \) – oversampling ratio;

\( \chi \) – constant factor characterizing kind of the input signal;

\( S \) – mean power of the input level;

\( fc \) – cut-off frequency of the input signal;

\( k_{\text{min}}, k_{\text{max}} \) – minimal, maximal quantization step size;

\( K_k = k_{\text{max}} / k_{\text{min}} \) – extension ratio of the quantization step sizes to achieve the desired dynamic range \( DR_k \) of the constant SNR value in modulations with constant \( f_s \);

\( B_{\text{max}} = \frac{f_s}{2fc}, \quad (B_{\text{min}} = \frac{f_s}{2fc}) \) – oversampling ratio of the maximum (minimum) sampling frequency \( DR_f = \sqrt{S^2} / \sqrt{S^1} \);

\( B_{\text{max}}, B_{\text{min}} = \frac{f_s}{2fc}, \quad \tau_{\text{min}}, \tau_{\text{max}} \) – the extension ratio of the sampling intervals to achieve the desired dynamic range \( DR_f \) of the constant SNR value in modulation with constant \( k \).

The knowledge of the sampling interval range \( K_f \) and the algorithm of the ANSDM delta converter allow finding the necessary number of the sampling intervals and their values that maximizes SNR in assumed \( DR_f \).

4. Design values of the sampling interval range \( K_f \) and quantization step size range \( K_k \) for ANSDM modulators

Basing on the equation (1, 1’) the dynamic range \( DR \) of ANSDM modulation may be written as:
Dependency (2) which shows the total dynamic range of the ANSDM is the product of the dynamic range both from sampling interval and step size adaptation. The total dynamic range of the ANSDM modulation can be calculated as a product of the dynamic range derived both from sampling frequency $DR_f$ and step size adaptation $DR_k$ [3, 4, 5, 6]. The total dynamic range of the ANSDM modulation is a very important performance metric. It shows decreasing in the indispensable number of the ANSDM adaptive parameters. As seen from (2) an infinite number of pairs of the coefficients $DR_k, DR_f$ that help to provide the equation (2) occurs. Only in practice, definite, chosen applications help to determine the useful pairs.

From equation (2) the following relation is obtained in the logarithmic measure:

$$DR_{[dB]} = DR_k_{[dB]} + DR_f_{[dB]}$$  (3)

The total dynamic range of the ANSDM ($DR_{[dB]}$) in dB is the sum of the dynamic range from step size changes ($DR_k$) and from sampling frequency changes ($DR_f$) [2, 3].

As the designing aspect is considered the required dynamic range $DR$ is given. Then the equations $K_k = f_1(DR, K_f, B_{min})$ and $K_f = f_2(DR, K_k, B_{min})$ have to be determined from (2). The special function $LambertW(-1, -x)$ [9, 10], could be used to calculate the $K_f$ value (from transcendental for $K_f$ equations 1, 1' and 2).

The solutions have a form [3, 4]:

$$K_k = \sqrt{DR} \left( \frac{\ln(B_{max}) + \ln(K_f)}{K_f \ln(2B_{max})} \right)$$  (4)

$$K_f = -\frac{\sqrt{DR}}{K_k \ln(2B_{max})} LambertW(-1, -K_k \ln(2B_{max})/2B_{max}\sqrt{DR})$$  (5)

Equation (8) has real solutions [5] when

$$0 < \frac{-\ln(2B_{max})}{2B_{max}\sqrt{DR}} < \frac{1}{e}$$  (6)

The sampling frequency ratio $K_f$ should have values resulting from (5), in order to keep the SNR value at the constant level, within the input power range $S_f/S_f$. The assumption of a greater value is not advisable, whereas a less value will not guarantee a constant level of SNR value within all input power range. From equations (4) and (5) it is seen that the $DR$ effectiveness of CFDM modulation is better than the NSDM.

5. Examples of determining the internal parameters of NSDM and ANSDM modulators based on the Maple platform

On the basis of the established order of external parameters calculation, rules were developed that allow analytical determination of the NSDM and ANSDM coder internal parameters. Due to the high complexity of the calculations, the ANSDMsoft program was developed to ensure the automation of calculations [7]. Two examples of it were shown in Fig. 4.

6. Examples of $DR$ determination using ANSDMsoft

The ANSDM conversion, allowing for the simultaneous adaptation of the quantization step size and the sampling interval, means that there is an infinite number of $DR_k, DR_f$ pairs to achieve the desired global dynamic range $DR$ [7].

Currently, a number of computer programs supporting symbolic mathematical calculations (CAS – Computer Algebra System) are available. The most popular are Mathematica by Wolfram Research, Mathcad by Mathsoft, and Maple by Maplesoft.

All computational works were carried out using the Maple environment. Maple allows to solve complex mathematical functions and display their results in a simple way. Most importantly, it supports the $LambertW$ function [8, 9], used in the computing of NSDM or ANSDM modulator parameters.

The developed tool includes interactive examples of CFDM, NSDM and ANSDM modulators parameter evaluation and their implementation (Fig. 5, 6).

Interactive parts of the spreadsheet have been built with the help of embedded components. These are simple GUI (Graphical user interface) elements such as Buttons, Sliders, Text area, Labels, and expandable Combo box [7].

After specifying the input data, the user receives information about the values of the possible dynamic range $DR$, which is also visualized using a graph.

The user has two sliders at his disposal, responsible for selecting the minimum and maximum sampling frequency (Example 1). The app does not allow you to set a value $F_{max} > F_{max}$. The change of the maximum sampling frequency entails a change in the available range of parameter $F_{max}$.
Figure 4. Examples of flow diagrams describing the sequence of determining the internal parameters of the NS-DM coder for various input data.

**Example 1**
Graphic illustration of the NSDM modulator dynamic range as a function of the minimum and maximum sampling frequency ($K_f = 50$, $K_k = 1$) [7].

**Example 2**
Graphic illustration of the ANSDM modulator dynamic range as a function of the minimum and maximum sampling frequency ($K_f = 10$, $K_k = 10$) [7].

![Flow diagram](image)

Figure 5. Determining the dynamic range for NSDM modulation.
For ANSDM modulation, in order to determine the dynamic range $DR$, the ranges of adapted parameters change $K_f$ and $K_k$ should be introduced.

The user receives information about the total dynamic range $DR$ and its division into ranges $DR_f$ and $DR_k$ and, resulting from adaptation $f_s$ and $k$.

7. Conclusions

Description of the dynamic range $DR$ of the delta converters with adaptive sampling has been worked out. The method of dynamic range $DR$ calculating based on special function Lambert $W$ was proposed. It was proved, that the total dynamic range $DR$ of delta converter with double adaptation (ANSDM) can be expressed, in decibels, as the sum of $DR_f$ coming from sampling rate and $DR_k$ coming from step size changes. The functional models, flow diagrams, as well as the original method of the internal parameters selection of the delta converters with adaptive sampling, have been proposed. The form of the Abate dependency (1) and (1’), describing the condition of the minimum quantization noise makes the special function Lambert $W$ extremely useful for determining the maximum and minimum sampling frequency.

On the basis of the established sequence of calculating internal parameters, some rules were developed that allow the analytical determination of all external parameters necessary for the correct operation of NSDM and ANSDM converters. They were used in the ANSDMsoft program.

**Author Contributions**

Conceptualization, Ryszard Golański; methodology, Ryszard Golański and Juliusz Godek; validation, Ryszard Golański and Juliusz Godek; formal analysis, Ryszard Golański; investigation, Ryszard Golański; resources, Ryszard Golański; data curation, Ryszard Golański; writing—original draft preparation, Ryszard Golański; writing—review and editing, Ryszard Golański; visualization, Ryszard Golański and Juliusz Godek; supervision, Ryszard Golański; funding acquisition, Ryszard Golański.

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